# Decoherence of a quantum harmonic oscillator monitored by a Bose-Einstein condensate 

S. Brouard, ${ }^{1,2, *}$ D. Alonso, ${ }^{2,3, \dagger}$ and D. Sokolovski ${ }^{4,5}$<br>${ }^{1}$ Departamento de Física Fundamental II, Universidad de La Laguna, La Laguna E-38204, Tenerife, Spain<br>${ }^{2}$ Instituto Universitario de Estudios Avanzados (IUdEA)<br>${ }^{3}$ Departamento de Física Fundamental y Experimental, Electrónica y Sistemas, Universidad de La Laguna, La Laguna E-38204, Tenerife, Spain<br>${ }^{4}$ Department of Chemical Physics, University of the Basque Country, Leioa, Spain<br>${ }^{5}$ IKERBASQUE, Basque Foundation for Science, E-48011, Bilbao, Spain

(Received 28 October 2010; published 22 July 2011)


#### Abstract

We investigate the dynamics of a quantum oscillator, whose evolution is monitored by a Bose-Einstein condensate (BEC) trapped in a symmetric double-well potential. It is demonstrated that the oscillator may experience various degrees of decoherence depending on the variable being measured and the state in which the BEC is prepared. These range from a "coherent" regime in which only the variances of the oscillator position and momentum are affected by measurement, to a slow (power-law) or rapid (Gaussian) decoherence of the mean values themselves.


DOI: 10.1103/PhysRevA.84.012114

## I. INTRODUCTION

In the past few years there has been much interest, both theoretical and experimental, in nanomechanical oscillators whose quantum behavior can be observed (measured) within the limits imposed by the uncertainty relations [1,2]. Typically, various degrees of coherent control over such oscillators can be achieved by incorporating them in hybrid devices involving superconducting microwave cavities [3], superconducting qubits [4,5], single-electron transistors [6,7], and point contacts (PCs) [8]. More recently, several schemes for coupling a quantum system to a Bose-Einstein condensate (BEC) have been proposed [9]. In the case of a measurement involving a PC in a large bias regime, interaction between an oscillator and the electron current damps the latter, leaving it in an equilibrium thermal state [10]. In this work we analyze a setup in which an oscillator is coupled to a BEC trapped in a symmetric double-well potential rather than to a PC. With the atomic current dependent on the oscillator coordinate, the BEC is able to monitor the oscillator evolution, at the cost of introducing decoherence to the oscillator dynamics. This decoherence is the main subject of this paper. We will show that, unlike in the case of a point contact, an oscillator monitored by a BEC does not, in general, reach a thermal equilibrium $[10,11]$ and may, in some cases, retain a degree of coherence, depending on the oscillator variable being monitored as well as on the initial state of the BEC. The absence of a transition to thermal equilibrium predicted, for example, for an oscillator coupled to a PC, is a consequence of the fact that a single energy level, rather than a broad energy band, is available for each tunneling boson. For recent relevant work on the types of decoherence possible in open systems we refer the reader to Ref. [12].

## II. THE DETECTOR MODEL

We consider a system described by the Hamiltonian that is a generalization of the "gatekeeper" model introduced in

[^0]PACS number(s): 03.65.Yz, 03.65.Ta, 03.75.Gg
Ref. [13], i.e., $N$ noninteracting [14] particles of a BEC trapped in a double-well potential coupled to a harmonic oscillator that is being monitored (we put $\hbar=1$ ),

$$
\begin{equation*}
H=H_{\mathrm{con}}+H_{\mathrm{osc}}+\delta \Omega A_{\mathrm{osc}} \otimes B_{\mathrm{con}} \tag{1}
\end{equation*}
$$

where

$$
\begin{gather*}
H_{\mathrm{con}}=\Omega_{0}\left(c_{L}^{\dagger} c_{L}+c_{R}^{\dagger} c_{R}\right)-\Omega_{1}\left(c_{L}^{\dagger} c_{R}+c_{R}^{\dagger} c_{L}\right) \\
H_{\mathrm{osc}}=\omega_{0}\left(a^{\dagger} a+\frac{1}{2}\right)  \tag{2}\\
B_{\mathrm{con}}=c_{L}^{\dagger} c_{R}+c_{R}^{\dagger} c_{L}
\end{gather*}
$$

and $c_{L}^{\dagger}\left(c_{R}^{\dagger}\right)$ are the creation operators for the particles of the condensate in the left (right) reservoir (Fig. 1). $a^{\dagger}(a)$ is the creation (annihilation) operator of a harmonic oscillator with frequency $\omega_{0}$ and mass $m, H_{\text {osc }}=P^{2} / 2 m+\frac{1}{2} m \omega_{0}^{2} X^{2}$. The oscillator is assumed to be charged so as to affect the barrier between the two wells. The operator $A_{\text {osc }}$ represents the variable that controls the tunneling rate between the wells, so that its evolution can be monitored by observing the atomic current or a change in the number of bosons in one of the reservoirs. Both couplings linear $\left(A_{\text {osc }} \sim X\right)$ and quadratic ( $A_{\text {osc }} \sim X^{2}$ ) in the oscillator coordinate are possible [15]. Coupled in such a manner, the BEC shares with a conventional von Neumann meter [16] the property that if prepared in a stationary state, it will remain in that state inducing an additional force on the measured oscillator [17].

With $H_{\text {con }}$ and $B_{\text {con }}$ commuting, $\left[H_{\text {con }}, B_{\text {con }}\right]=0$, such states are easily found to be

$$
\begin{equation*}
\left|\tilde{\phi}_{n}\right\rangle=\frac{\left(c_{L}^{\dagger}+c_{R}^{\dagger}\right)^{N-n}\left(c_{L}^{\dagger}-c_{R}^{\dagger}\right)^{n}}{\sqrt{2^{N}(N-n)!n!}}|0\rangle_{\text {con }}, \quad(n=0, \ldots, N) \tag{3}
\end{equation*}
$$

where the vacuum $|0\rangle_{\text {con }}$ corresponds to no bosons in the condensate. Assuming that the oscillator and the BEC are prepared in a product state, $\rho(0)=\rho_{\text {osc }}(0) \otimes \rho_{\text {con }}(0)$, and noting that $B_{\text {con }}\left|\tilde{\phi}_{n}\right\rangle=(N-2 n)\left|\tilde{\phi}_{n}\right\rangle$, we find the state of the


FIG. 1. Double-well potential containing the $N$ bosons that may tunnel from one well to the other. The flow of bosons is modulated by some function of the position of the oscillator.
monitored oscillator at a time $t$ by tracing out the BEC degrees of freedom,

$$
\begin{align*}
\rho_{\mathrm{osc}}(t) & =\operatorname{Tr}_{\mathrm{con}}\left[e^{-i H t} \rho(t) e^{i H t}\right] \\
& =\sum_{n}\left\langle\tilde{\phi}_{n}\right| e^{-i H t} \rho(t) e^{i H t}\left|\tilde{\phi}_{n}\right\rangle \\
& =\sum_{n} P_{n} \rho_{\mathrm{osc}}^{(n)}(t), \tag{4}
\end{align*}
$$

with

$$
\begin{gather*}
P_{n} \equiv\left\langle\tilde{\phi}_{n}\right| \rho_{\mathrm{con}}(0)\left|\tilde{\phi}_{n}\right\rangle \\
\rho_{\mathrm{osc}}^{(n)}(t) \equiv e^{-i \mathcal{H}_{\mathrm{osc}}\left(\epsilon_{n}\right) t} \rho_{\mathrm{osc}}(0) e^{i \mathcal{H}_{\mathrm{osc}}\left(\epsilon_{n}\right) t}, \tag{5}
\end{gather*}
$$

$\mathcal{H}_{\mathrm{osc}}(\epsilon) \equiv H_{\mathrm{osc}}+\epsilon A_{\mathrm{osc}}$, and $\epsilon_{n} \equiv \delta \Omega(N-2 n)$.
Thus, $\rho_{\text {osc }}(t)$ is an incoherent superposition of the states obtained by evolving $\rho_{\text {osc }}(0)$ with the family of Hamiltonians $\mathcal{H}_{\text {osc }}\left(\epsilon_{n}\right), n=0,1, \ldots, N$, weighted by the probabilities $P_{n}$ to find the BEC in the state $\left|\tilde{\phi}_{n}\right\rangle$. Accordingly, at a time $t$, the expectation value of an oscillator variable represented by an operator $O_{\text {osc }}$ is given by the sum

$$
\begin{equation*}
\left\langle O_{\mathrm{osc}}(t)\right\rangle=\sum_{n} P_{n} \operatorname{Tr}_{\mathrm{osc}}\left[\rho_{\mathrm{osc}}^{(n)}(t) O_{\mathrm{osc}}\right] . \tag{6}
\end{equation*}
$$

Following Ref. [13], we take the limit in which the number of atoms becomes large, while the coupling between the oscillator and each individual atom is reduced, namely

$$
\begin{equation*}
N \rightarrow \infty, \quad \delta \Omega \rightarrow 0, \quad \delta \Omega \sqrt{N}=\kappa \tag{7}
\end{equation*}
$$

and choose $\Omega_{1}=0$ so as to exclude a constant background current [20]. The conditions of Eq. (7) ensure that a macroscopic atomic current flows from the left to the right well, as the recurrence time of the condensate greatly exceeds the duration of the measurement. Thus, the atoms are not going to return to their initial state in the foreseeable future, i.e., the BEC becomes an irreversible meter [13,21]. As discussed in Ref. [13], for a condensate with a large but finite number of atoms the oscillator begins to be affected by the size of the condensate at times of the order of the Poincaire recurrence time (Rabi period) of the latter, i.e., when the escape of the atoms into the right well can no longer be considered irreversible [22].

Replacing sums by integrals, $2 \delta \Omega \sum_{n} \rightarrow \int_{-\infty}^{\infty} d \epsilon$, we rewrite Eq. (4) as

$$
\begin{equation*}
\rho_{\mathrm{osc}}(t)=\int_{-\infty}^{\infty} d \epsilon P(\epsilon) e^{-i \mathcal{H}_{\mathrm{osc}}(\epsilon) t} \rho_{\mathrm{osc}}(0) e^{i \mathcal{H}_{\mathrm{osc}}(\epsilon) t} \tag{8}
\end{equation*}
$$

where $P\left(\epsilon_{n}\right) \equiv P_{n} /(2 \delta \Omega)$. Then, separating stationary and time-dependent parts,

$$
\begin{align*}
\rho_{\mathrm{osc}}(t)= & \int_{-\infty}^{\infty} d \epsilon P(\epsilon)\left\{\sum_{i} \operatorname{big}\left\langle\psi_{i}^{\epsilon}\right| \rho_{\mathrm{osc}}(0)\left|\psi_{i}^{\epsilon}\right\rangle\left|\psi_{i}^{\epsilon}\right\rangle\left\langle\psi_{i}^{\epsilon}\right|\right. \\
& \left.+\sum_{i \neq j} e^{-i\left(E_{i}^{\epsilon}-E_{j}^{\epsilon}\right) t}\left\langle\psi_{i}^{\epsilon}\right| \rho_{\mathrm{osc}}(0)\left|\psi_{j}^{\epsilon}\right\rangle\left|\psi_{i}^{\epsilon}\right\rangle\left\langle\psi_{j}^{\epsilon}\right|\right\} \tag{9}
\end{align*}
$$

where $E_{i}^{\epsilon}$ and $\left|\psi_{i}^{\epsilon}\right\rangle$ are the eigenvalues and eigenvectors, respectively, of the Hamiltonian $\mathcal{H}_{\text {osc }}(\epsilon)$. The long-time behavior of $\rho_{\text {osc }}(t)$ now depends on the spectra $E_{i}^{\epsilon}$. Indeed, for $E_{i}^{\epsilon}-E_{j}^{\epsilon} \neq \operatorname{const}(\epsilon)$ rapidly oscillating exponentials will cause the second term in Eq. (9) to vanish, so that $\rho_{\text {osc }}(t)$ [and with it the averages of Eq. (6)] will tend to stationary values as $t \rightarrow \infty$. Without such a cancellation, the oscillator will not be able to reach a steady state no matter how long one waits.

Consider further a BEC initially localized in the left well, $\rho_{\text {con }}(0)=\left|\phi_{0}\right\rangle\left\langle\phi_{0}\right|$, where $\left|\phi_{0}\right\rangle=\left(c_{L}^{\dagger}\right)^{N}|0\rangle_{\text {con }} / \sqrt{N!}$. For the probability weights $P_{n}$ in Eqs. (4) and (6) we have

$$
\begin{equation*}
P_{n}=\frac{N!}{2^{N}(N-n)!n!}, \tag{10}
\end{equation*}
$$

and applying the Stirling formula in the limit of Eq. (7) yields $P(\epsilon)=\left(2 \pi \kappa^{2}\right)^{-1 / 2} \exp \left(-\epsilon^{2} / 2 \kappa^{2}\right)$. Rather than analyze the density matrix of Eq. (4), it is convenient to consider the mean position and momentum of the monitored oscillator, together with their variances, thus choosing the operator $O_{\text {osc }}$ in Eq. (6) to be $X, X^{2}, P$, or $P^{2}$.

## III. MONITORING OF A ONE-DIMENSIONAL HARMONIC OSCILLATOR

## A. Monitoring position: Coherent motion with "breathing"

With $A_{\mathrm{osc}} \equiv a^{\dagger}+a=\sqrt{2 m \omega_{0}} X \equiv X / X_{0}$, the BEC monitors oscillator's position $X$, and we need to consider motion in a family of harmonic potentials shifted relative to the original one by $\delta x_{n} \equiv \sqrt{\frac{2}{m \omega_{0}}} \frac{\epsilon_{n}}{\omega_{0}}, n=0,1, \ldots, N$,

$$
\begin{equation*}
V_{n}(x)=m \omega_{0}^{2}\left(x+\delta x_{n}\right)^{2} / 2-m \omega_{0}^{2} \delta x_{n}^{2} / 2 \tag{11}
\end{equation*}
$$

Thus, the energy differences in Eq. (9) are independent of $\epsilon$, $E_{i}^{\epsilon}-E_{j}^{\epsilon}=\omega_{0}(i-j)$, and no steady state can be reached. We note further that since $\mathcal{H}_{\text {osc }}(\epsilon)$ in Eq. (5) remains quadratic in both $X$ and $P$, equations of motion for the five operators $X, P$, $X^{2}, P^{2}$, and $(X P)_{s} \equiv(X P+P X) / 2$ form a closed system,

$$
\begin{gathered}
\frac{d\langle X(t)\rangle_{n}}{d t}=\frac{1}{m}\langle P(t)\rangle_{n} \\
\frac{d\langle P(t)\rangle_{n}}{d t}=-m \omega_{0}^{2}\langle X(t)\rangle_{n}-\sqrt{2 m \omega_{0}} \epsilon_{n} \\
\frac{d\left\langle X^{2}(t)\right\rangle_{n}}{d t}=\frac{2}{m}\left\langle(X P)_{s}(t)\right\rangle_{n} \\
\frac{d\left\langle P^{2}(t)\right\rangle_{n}}{d t}=-2 m \omega_{0}^{2}\left\langle(X P)_{s}(t)\right\rangle_{n}-2 \sqrt{2 m \omega_{0}} \epsilon_{n}\langle P(t)\rangle_{n}
\end{gathered}
$$

$$
\begin{align*}
\frac{d\left\langle(X P)_{s}(t)\right\rangle_{n}}{d t}= & \frac{1}{m}\left\langle P^{2}(t)\right\rangle_{n}-m \omega_{0}^{2}\left\langle X^{2}(t)\right\rangle_{n} \\
& -\sqrt{2 m \omega_{0}} \epsilon_{n}\langle X(t)\rangle_{n} \tag{12}
\end{align*}
$$

which can be solved for each value $\epsilon_{n}$. Then, for $\langle X(t)\rangle_{n}$ and $\langle P(t)\rangle_{n}$, one obtains

$$
\begin{align*}
\langle X(t)\rangle_{n}= & \left(\langle X(0)\rangle+\sqrt{\frac{2}{m \omega_{0}}} \frac{\epsilon_{n}}{\omega_{0}}\right) \cos \left(\omega_{0} t\right) \\
& +\frac{\langle P(0)\rangle}{m \omega_{0}} \sin \left(\omega_{0} t\right)-\sqrt{\frac{2}{m \omega_{0}}} \frac{\epsilon_{n}}{\omega_{0}} \\
\langle P(t)\rangle_{n}= & -m \omega_{0}\left(\langle X(0)\rangle+\sqrt{\frac{2}{m \omega_{0}}} \frac{\epsilon_{n}}{\omega_{0}}\right) \sin \left(\omega_{0} t\right) \\
& +\langle P(0)\rangle \cos \left(\omega_{0} t\right), \tag{13}
\end{align*}
$$

where $\left\langle O_{\text {osc }}(0)\right\rangle=\operatorname{Tr}_{\text {osc }}\left[O_{\text {osc }} \rho_{\text {osc }}(0)\right]$ is the expectation value of $O_{\text {osc }}$ in the initial oscillator state. Replacing solutions of Eq. (13) in the last three equations in Eq. (12) and averaging with the probabilities $P_{n}$ then yields

$$
\begin{gather*}
\frac{d\langle X(t)\rangle}{d t}=\frac{1}{m}\langle P(t)\rangle \\
\frac{d\langle P(t)\rangle}{d t}=-m \omega_{0}^{2}\langle X(t)\rangle \\
\frac{d\left\langle X^{2}(t)\right\rangle}{d t}= \\
\frac{d\left\langle P^{2}(t)\right\rangle}{d t}=-2 m \omega_{0}^{2}\left\langle(X P)_{s}(t)\right\rangle+4 m \kappa^{2} \sin \left(\omega_{0} t\right) \\
\frac{d\left\langle(X P)_{s}(t)\right\rangle}{d t}= \\
\frac{1}{m}\left\langle P^{2}(t)\right\rangle-m \omega_{0}^{2}\left\langle X^{2}(t)\right\rangle  \tag{14}\\
-\frac{2 \kappa^{2}}{\omega_{0}}\left[\cos \left(\omega_{0} t\right)-1\right]
\end{gather*}
$$

where $\sum_{n} P_{n} \epsilon_{n}=0$ and $\sum_{n} P_{n} \epsilon_{n}^{2}=\kappa^{2}$ have been used. From the first two equations, we find the mean values of both the coordinate and the momentum unchanged by the presence of the BEC, $\langle X(t)\rangle=\langle X(t)\rangle_{\text {free }}$ and $\langle P(t)\rangle=\langle P(t)\rangle_{\text {free }}$, where the subscript "free" refers to an oscillator uncoupled from the BEC $(\kappa=0)$. This does not, however, imply that the BEC has no effect on the dynamics of the oscillator. Indeed, calculating the variances we find

$$
\begin{gather*}
(\Delta X)^{2}=\left\langle X^{2}\right\rangle-\langle X\rangle^{2}=(\Delta X)_{\text {free }}^{2}+4 \sigma_{X}^{2} \sin ^{4}\left(\omega_{0} t / 2\right) \\
(\Delta P)^{2}=\left\langle P^{2}\right\rangle-\langle P\rangle^{2}=(\Delta P)_{\text {free }}^{2}+\sigma_{P}^{2} \sin ^{2}\left(\omega_{0} t\right) \tag{15}
\end{gather*}
$$

where $\sigma_{X} \equiv \sqrt{\frac{2}{m \omega_{0}}} \frac{\kappa}{\omega_{0}}$ and $\sigma_{P} \equiv \sqrt{2 m \omega_{0}} \frac{\kappa}{\omega_{0}}$. Thus, while $\langle X(t)\rangle$ and $\langle P(t)\rangle$ follow their unperturbed trajectories, the widths of the corresponding distributions "breath," first increasing and then decreasing again. Figure 2 shows the dynamics of the corresponding mean values and variances for an oscillator prepared in a coherent state (minimal Gaussian wavepacket) $\left\langle x \mid \psi_{\text {osc }}(0)\right\rangle=$ $\left(m \omega_{0} / \pi\right)^{1 / 4} e^{-m \omega_{0}[x-\langle X(0)\rangle]^{2} / 2} e^{i\langle P(0)\rangle x}$ with $\kappa^{2} / \omega_{0}^{2}=25$. Note that $\Delta X(t)$ recovers its original value after every period $T=2 \pi / \omega_{0}$ as the oscillator returns to its initial state in each $V_{n}(x)$. The momentum variance $\Delta P(t)$ does so also after every


FIG. 2. (Color online) Coherent motion ( $X$ is monitored) of a coherent initial oscillator state with $\langle X(0)\rangle / X_{0}=0,\langle P(0)\rangle / P_{0}=2$; $X_{0} \equiv\left(2 m \omega_{0}\right)^{-1 / 2}, P_{0} \equiv\left(m \omega_{0} / 2\right)^{1 / 2}$. (a) Mean position $\langle X(t)\rangle / X_{0}$ (thick solid) vs. $\omega_{0} t$. Also shown are $[\langle X(t)\rangle \pm \Delta X] / X_{0}$ (solid) and $\left[\langle X(t)\rangle \pm \Delta X_{\text {free }}\right] / X_{0}$ (dashed); (b) Mean momentum $\langle P(t)\rangle / P_{0}$ (thick solid) vs. $\omega_{0} t$. Also shown are $[\langle P(t)\rangle \pm \Delta P] / P_{0}$ (solid) and $\left[\langle P(t)\rangle \pm \Delta P_{\text {free }}\right] / P_{0}$ (dashed). Inset: closed phase space trajectory traced by the mean momentum and position, $\langle P(t)\rangle / P_{0}$ vs. $\langle X(t)\rangle / X_{0}$.
half-period, when the shape of the original wavepacket is restored but the position of its center is reflected with respect to the origin of each $V_{n}$, i.e., when $\Delta X(t)$ reaches its maximum value.

For other initial states, Eqs. (15) remain valid and changes only occur in the explicit expression taken by $(\Delta X)_{\text {free }}^{2}$ and $(\Delta P)_{\text {free }}^{2}$.

## B. Monitoring $\boldsymbol{x}^{\mathbf{2}}$ : Gaussian decoherence

With $A_{\text {osc }} \equiv\left(a^{\dagger}+a\right)^{2}=X^{2} / X_{0}^{2}$, the BEC monitors the square of the oscillator's position, $X^{2}$, and we need to consider motion in a family of harmonic potentials with the same origin, but with different frequencies, $\omega_{n}=\sqrt{\omega_{0}^{2}+4 \epsilon_{n} \omega_{0}}$, $n=0,1, \ldots, N$,

$$
\begin{equation*}
V_{n}(x)=\frac{1}{2} m\left(\omega_{0}^{2}+4 \epsilon_{n} \omega_{0}\right) x^{2}=\frac{1}{2} m \omega_{n}^{2} X^{2} . \tag{16}
\end{equation*}
$$

Now $E_{i}^{\epsilon}-E_{j}^{\epsilon}=(i-j) \sqrt{\omega_{0}^{2}+4 \epsilon \omega_{0}} \neq$ const $(\epsilon)$, and we expect that in the irreversible limit of Eq. (7) oscillator will undergo a relaxation to the steady state given by the first term in Eq. (9). Solving equations of motion for each oscillator frequency $\omega_{n}$ and averaging with the probabilities of Eq. (10), we find

$$
\begin{aligned}
\langle X(t)\rangle= & \sum_{n} P_{n}\left\{\langle X(0)\rangle \cos \left(\omega_{n} t\right)+\frac{\langle P(0)\rangle}{m \omega_{n}} \sin \left(\omega_{n} t\right)\right\} \\
\left\langle X^{2}(t)\right\rangle= & \sum_{n} P_{n}\left\{\frac{\left\langle(X P)_{s}(0)\right\rangle}{m \omega_{n}} \sin \left(2 \omega_{n} t\right)+\frac{\left\langle P^{2}(0)\right\rangle}{2 m^{2} \omega_{n}^{2}}\right. \\
& \left.\times\left[1-\cos \left(2 \omega_{n} t\right)\right]+\frac{\left\langle X^{2}(0)\right\rangle}{2}\left[1+\cos \left(2 \omega_{n} t\right)\right]\right\}
\end{aligned}
$$

$$
\begin{align*}
\langle P(t)\rangle= & \sum_{n} P_{n}\left\{\langle P(0)\rangle \cos \left(\omega_{n} t\right)-m \omega_{n}\langle X(0)\rangle \sin \left(\omega_{n} t\right)\right\} \\
\left\langle P^{2}(t)\right\rangle= & \sum_{n} P_{n}\left\{-m \omega_{n}\left\langle(X P)_{s}(0)\right\rangle \sin \left(2 \omega_{n} t\right)+\frac{\left\langle P^{2}(0)\right\rangle}{2}\right. \\
& \left.\times\left[1+\cos \left(2 \omega_{n} t\right)\right]+\frac{m^{2} \omega_{n}^{2}}{2}\left\langle X^{2}(0)\right\rangle\left[1-\cos \left(2 \omega_{n} t\right)\right]\right\} \tag{17}
\end{align*}
$$

We note that for $\epsilon_{n}<-\omega_{0} / 4$, $\omega_{n}$ becomes imaginary as the interaction turns the oscillator potential into a parabolic repeller, leading to the break up of the system. Choosing the interaction to be small enough to neglect the possibility of breakup and taking the limit of Eq. (7), we replace the sums in Eqs. (17) by integrals to obtain

$$
\begin{gather*}
\langle X(t)\rangle=\langle X(0)\rangle \operatorname{Re}\left[f_{0}(t)\right]+\frac{\langle P(0)\rangle}{m \omega_{0}} \operatorname{Im}\left[f_{1}(t)\right], \\
\langle P(t)\rangle=\langle P(0)\rangle \operatorname{Re}\left[f_{0}(t)\right]-m \omega_{0}\langle X(0)\rangle \operatorname{Im}\left[f_{-1}(t)\right], \tag{18}
\end{gather*}
$$

with

$$
\begin{equation*}
f_{\beta}(t) \equiv \frac{1}{\sqrt{2 \pi \sigma^{2}}} \int_{-1}^{\infty} d z e^{-z^{2} / 2 \sigma^{2}} \frac{e^{i \omega_{0} t \sqrt{1+z}}}{(1+z)^{\beta / 2}} \tag{19}
\end{equation*}
$$

where $\sigma=4 \kappa / \omega_{0}$. Similar expressions can be easily obtained for $\left\langle X^{2}(t)\right\rangle$ and $\left\langle P^{2}(t)\right\rangle$,

$$
\begin{align*}
\left\langle X^{2}(t)\right\rangle= & \frac{\left\langle(X P)_{s}(0)\right\rangle}{m \omega_{0}} \operatorname{Im}\left[f_{1}(2 t)\right]+\frac{\left\langle P^{2}(0)\right\rangle}{2 m^{2} \omega_{0}^{2}}\left\{1-\operatorname{Re}\left[f_{2}(2 t)\right]\right\} \\
& +\frac{\left\langle X^{2}(0)\right\rangle}{2}\left\{1+\operatorname{Re}\left[f_{0}(2 t)\right]\right\} \\
\left\langle P^{2}(t)\right\rangle= & -m \omega_{0}\left\langle(X P)_{s}(0)\right\rangle \operatorname{Im}\left[f_{-1}(2 t)\right] \\
& +\frac{\left\langle P^{2}(0)\right\rangle}{2}\left\{1+\operatorname{Re}\left[f_{0}(2 t)\right]\right\} \\
& +\frac{m^{2} \omega_{0}^{2}}{2}\left\langle X^{2}(0)\right\rangle\left\{1-\operatorname{Re}\left[f_{-2}(2 t)\right]\right\} \tag{20}
\end{align*}
$$

where $\sum_{n} P_{n} / \omega_{n}^{2} \simeq 1 / \omega_{0}^{2}$ was used since $\kappa \ll \omega_{0}$. For a weak coupling, $\sigma \ll 1$, and times not exceeding $1 / \omega_{0} \sigma^{2}$, the exponent in Eq. (19) can be expanded up to the second order in $z$. Replacing the lower limit of integration by $-\infty$ and evaluating Gaussian integrals yields

$$
\begin{equation*}
f_{\beta}(t)=\sqrt{\frac{1}{1+i \omega_{0} \sigma^{2} t / 4}} e^{-\omega_{0}^{2} \sigma^{2} t^{2} /\left(8+2 i \sigma^{2} \omega_{0} t\right)} e^{i \omega_{0} t}+O(\sigma) \tag{21}
\end{equation*}
$$

It is readily seen that, irrespective of the value of $\beta$, $\left|f_{\beta}(t)\right|$ decays on a time scale $T \equiv 1 /\left(\omega_{0} \sigma\right) \approx \kappa^{-1}$ so that for $t \simeq \omega_{0} / \kappa^{2} \gg T$ both the mean position $\langle X(t)\rangle$ and the mean momentum $\langle P(t)\rangle$ will have decayed to zero, regardless of their initial values, the decay being Gaussian in time.

Figure 3 illustrates Gaussian decoherence for the same initial coherent state as in Fig. 2 and for $N=100$ and $\delta \Omega / \omega_{0}=0.0024$. Again, the expressions in Eqs. (17) and


FIG. 3. (Color online) Gaussian decoherence ( $X^{2}$ is monitored) of a coherent initial oscillator state with $\langle X(0)\rangle / X_{0}=0,\langle P(0)\rangle / P_{0}=$ 2; $X_{0} \equiv\left(2 m \omega_{0}\right)^{-1 / 2}, P_{0} \equiv\left(m \omega_{0} / 2\right)^{1 / 2}$. (a) Mean position $\langle X(t)\rangle / X_{0}$ (thick solid) vs. $\omega_{0} t$. Also shown are $[\langle X(t)\rangle \pm \Delta X] / X_{0}$ (dashed); (b) Mean momentum $\langle P(t)\rangle / P_{0}$ (thick solid) vs. $\omega_{0} t$. Also shown are $[\langle P(t)\rangle \pm \Delta P] / P_{0}$ (dashed). Inset: decaying phase space trajectory traced by the mean momentum and position, $\langle P(t)\rangle / P_{0}$ vs. $\langle X(t)\rangle / X_{0}$ as given by Eqs. (17) (thick solid) and (18)-(21) (dashed).
(18) are formally independent of the initial state chosen for the harmonic oscillator.

We note [and this is a general effect of a weak coupling, $\kappa \ll \omega_{0}$, acting over a long time $t \gg T$; c.f. Eq. (17)] that in the final steady state of the oscillator, the initial energy is shared equally between its kinetic and potential components; i.e., for $t \gg T$ we have

$$
\begin{equation*}
\frac{\left\langle P^{2}(t)\right\rangle}{2 m}=\frac{1}{2} m \sum_{n} P_{n} \omega_{n}^{2}\left\langle X^{2}(t)\right\rangle_{n}=\frac{\left\langle H_{\mathrm{osc}}(0)\right\rangle}{2} \tag{22}
\end{equation*}
$$

## C. Power law decoherence

Other types of decoherence are possible with different choices of the initial state of the BEC. For example, a powerlaw decoherence can be achieved by replacing in Eq. (19) the smooth Gaussian factor by a discontinuous one. Thus, choosing

$$
\begin{equation*}
P_{n} \equiv\left|\left\langle\tilde{\phi}_{n} \mid \phi_{0}\right\rangle\right|^{2}=e^{-\alpha \omega_{n}} / \sum_{n \geqslant N / 2}^{N} e^{-\alpha \omega_{n}} \tag{23}
\end{equation*}
$$

for $n \geqslant N / 2$ and zero otherwise and taking the limit of Eq. (7) for a state with $\langle X(0)\rangle=0$ yields

$$
\begin{equation*}
\langle X(t)\rangle=\langle P(0)\rangle \frac{\alpha \sin \left(\omega_{0} t\right)+t \cos \left(\omega_{0} t\right)}{m\left(\alpha \omega_{0}+1\right)\left(1+t^{2} / \alpha^{2}\right)}, \tag{24}
\end{equation*}
$$

so that for $t \gg \alpha,\langle X(t)\rangle$ tends to zero as $1 / t$.
In summary, in a hybrid setup involving an oscillator and a BEC in a symmetric double-well potential, we have an example of an exactly solvable detector model demonstrating
nontrivial dynamics. In the irreversible limit, the meter provides unidirectional macroscopic atomic current whose magnitude depends on the oscillator's position. Unlike in the case of a point contact, the measurement does not lead to universal damping of the oscillator and eventual thermalization of its initial state. Rather, depending on the oscillator variable being monitored as well as on the initial state of the BEC, the oscillator may or may not undergo relaxation to a steady state and retain a degree of initial coherence. Such a behavior is a consequence of the fact that a single energy level, rather
than a broad energy band, is available for each tunneling boson.

## ACKNOWLEDGMENTS

We are grateful to Shmuel Gurvitz for useful discussions. Two of us (S.B. and D.A.) acknowledge financial support provided by Ministerio de Educación y Ciencia, Spain (Grant Nos. FIS2007-64018 and FIS2010-19998). D.A. thanks E. Sentís Rodríguez for encouragement.
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[^0]:    *sbrouard@ull.es
    ${ }^{\dagger}$ dalonso@ull.es

