Statistical Physics II.
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Degenerate Fermi Gas
White dwarfs

- **In this section we shall study a simple model of White Dwarfs.**

  - Some numbers

```
<< PhysicalConstants`
<< Units`
```

```
StarDensity = 10^7 Gram / Centimeter^3
```

$\frac{10\,000\,000\ \text{Gram}}{\text{Centimeter}^3}$

```
Sm = CGS[1.9891 × 10^30 Kilogram]
```

1.9891×10^{33} Gram

```
Tc = 10^7 Kelvin
```

10 000 000 Kelvin

```
kB = BoltzmannConstant
```

$\frac{1.38065\times10^{-23}\ \text{Joule}}{\text{Kelvin}}$

  - The thermal energy E_T corresponding to the temperature T_c

```
ET = Convert[kB Tc, ElectronVolt]
```

861.734 ElectronVolt

- This energy is much higher than the Helium ionization energy

```
ElementData[2, "IonizationEnergies", "Units"]
Io = ElementData[2, "IonizationEnergies"]
```

```
KilojoulesPerMole
```

```
{2372.3, 5250.5}
```

```
IO1 = SI[Io 1000 Joule / Mole]
```

$\{3.9393\times10^{-18}$ Joule, 8.71867×10^{-18} Joule$\}$

```
Convert[IO1[[1]], ElectronVolt]
Convert[IO1[[2]], ElectronVolt]
```

```
24.5872 ElectronVolt
```

```
54.4176 ElectronVolt
```

- Now you can compare the to energies!

The Fermi Energy

- Helium Mass

```
ElementData[He, "Mass"]
```

```
HeMass = 4 / AvogadroConstant Gram / Mole
```

6.64216×10^{-24} Gram

```
ElectronDensity = SI[2 StarDensity / HeMass]
```

$$\frac{3.01107\times10^{36}}{\text{Meter}^3}$$

```
ℏ = PlanckConstantReduced
mₑ = ElectronMass
```

$$\epsilon_F = \frac{\hbar^2}{2\, m_e} \left(3\, \pi^2\, \text{ElectronDensity}\right)^{2/3} \text{ /. } \left\{\text{Joule} \to \text{Kilogram Meter}^2 \big/ \text{Second}^2,\ \left(\frac{1}{\text{Meter}^3}\right)^{2/3} \to 1 \big/ \text{Meter}^2\right\}$$

$$T_F = \% / k_B \text{ /. } \left\{\text{Joule} \to \text{Kilogram Meter}^2 \big/ \text{Second}^2\right\}$$

$$\text{Convert}[\epsilon_F,\ \text{ElectronVolt}]$$

$$\frac{1.21823 \times 10^{-13}\ \text{Kilogram Meter}^2}{\text{Second}^2}$$

8.82357×10^{9} Kelvin

760 357. ElectronVolt

- We see that T_F / T_c is such that we can consider that the electron Fermi gas is in its groud state

T_F / T_c

882.357

Estimation os Chandrasekhar's critical Mass

```
h = PlanckConstantReduced
g = GravitationalConstant
mp = ProtonMass
c = SpeedOfLight
Sm = SI[SolarMass]
```

$$factor = \frac{8\, mp}{9\, \pi};$$

$$M_0 = \left(\frac{27\, \pi}{64\, \alpha}\right)^{3/2} \left(\frac{h\, c}{g\, mp^2}\right)^{3/2} \text{/. \{Newton} \to \text{Joule / Meter\} /. \{Joule} \to \text{Kilogram Meter}^2 / \text{Second}^2\}$$

1.05457×10^{-34} Joule Second

$$\frac{6.67428 \times 10^{-11}\ \text{Meter}^2\ \text{Newton}}{\text{Kilogram}^2}$$

1.67262×10^{-27} Kilogram

$$\frac{299\,792\,458\ \text{Meter}}{\text{Second}}$$

1.9891×10^{30} Kilogram

$$3.3616 \times 10^{57} \left(\frac{1}{\alpha}\right)^{3/2}$$

M_0 factor / Sm /. {$\alpha \to 1$}
M_0 factor / Sm /. {$\alpha \to 2/3$}
M_0 factor / Sm /. {$\alpha \to 1.1$}

0.799806

1.46934

0.693259

```
SolarMass
```

SolarMass