Statistical Physics II.

Some approximations. Bose and Fermi ideal gas

```
TraditionalForm[-Sum[Log[1 - z Exp[-β ϵp]], p]]
TraditionalForm[Sum[Log[1 + z Exp[-β ϵp]], p]]
```

$$-\sum_p \log\left(1 - z\, e^{-\beta \epsilon_p}\right)$$

$$\sum_p \log\left(z\, e^{-\beta \epsilon_p} + 1\right)$$

Momentum grid, h = 1, m = 1, K_B=1

- **The Fermi case: $\sum_p \log\left(z\, e^{-\beta \epsilon_p} + 1\right)$**

```
a = 60; l = 5; T = 3; h = 1; β = (1 / T);
Energy = Flatten[Table[(h/l)^2 (n[1]^2 + n[2]^2 + n[3]^2) 0.5,
    {n[1], -a, a}, {n[2], -a, a}, {n[3], -a, a}]];
Canonical = Exp[-β Energy];
GCanonical = Log[1. + Canonical];
```

- **The sum over momenta**

```
Tr[GCanonical]
```

```
8871.17
```

- **The Integral approximation**

```
(4. π l^3)/h^3 NIntegrate[p^2 Log[1 + Exp[-p^2 0.5 β]], {p, 0, Infinity}]
```

```
8871.17
```

- **The Bose case:** $-\sum_p \log\left(1 - z\, e^{-\beta \epsilon_p}\right)$

```
z = 1;
GCanonical = Log[1 - z Canonical];
Position[1 - Canonical, 0]
Tr[GCanonical]
Sort[GCanonical];
Tr[Drop[%, -1]]
```

{{885 781}}

−∞

−13 715.7

$$\frac{4.\ \pi\ l^3}{h^3}\ \text{NIntegrate}\left[p^2\ \text{Log}\left[1 - z\ \text{Exp}\left[-p^2\ 0.5\ \beta\right]\right],\ \{p,\ 0,\ \text{Infinity}\}\right]$$

−13 723.